Physics PhD qualifying examinations

1985-1986

Columbia University

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COLUMBIA

DEPARTMENT OF PHYSICS

Ph.D. Qualifying Examination

CLASSICAL PHYSICS

Closed Book - Four Hours

ANSWER EACH QUESTION IN A SEPARATE BOOK AND MARK THE QUESTION NUMBER AND YOUR NAME CLEARLY ON EACH BOOK.

PART I - MECHANICS

Answer 3 out of 5 questions (1-5)

1. Consider a particle of mass m moving in two dimensions in a potential

$$V(x,y) = -\tfrac{1}{2}kx^2 + \tfrac{1}{2}\lambda_0 x^2 y^2 + \tfrac{1}{4}\lambda_1 x^4 \qquad (k, \lambda_0, \lambda_1 > 0)$$

(a) At what point (x_0, y_0) is the particle in stable equilibrium?

(b) Give the Lagrangian appropriate for small oscillations about this equilibrium position.

(c) What are the normal frequencies of vibration in (b)?

2. Consider a particle of mass m moving in a plane under a central force

$$F(r) = \frac{-k}{r^2} + \frac{k'}{r^3} \qquad \text{(assume } k > 0\text{)}$$

(a) What is the Lagrangian for this system in terms of the polar coordinates r, θ and their velocities?

(b) Write down the equations of motion for r and θ, and show that the orbital angular momentum (ℓ) is a constant of the motion.

(c) Assume that $\ell^2 > -mk'$. Find the equation for the orbit (i.e. r as a function of θ.)

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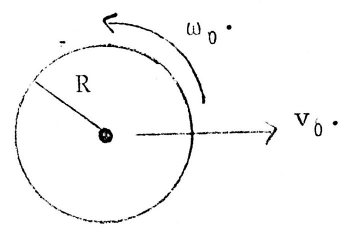
3. A wheel of mass M and radius R is projected along a horizontal surface with an initial linear velocity v_0 and with an initial angular velocity ω_0, so it starts sliding along the surface (ω_0 tends to produce rolling in the direction opposite to v_0.) Let the coefficient of friction between the wheel and the surface be μ.

(a) How long is it till the sliding ceases?

(b) What is the velocity of the center of mass of the wheel at the time when the slipping stops?



4. A perfectly smooth horizontal disk is rotating with an angular velocity ω about a vertical axis passing through its center. A person on the disk at a distance R from the origin gives a perfectly smooth coin (negligible size) of mass m a push directly at the origin. This push gives it an initial velocity v relative to the disk. Show that the motion for a time t (such that $(\omega t)^2$ is negligible) appears to the person on the disk to be a parabola, and give the equation of the parabola.

5. A particle under the action of gravity slides on the inside of a smooth paraboloid of revolution whose axis is vertical. Using the distance from the axis r and the azimuthal angle ϕ as generalized coordinates, find:

(a) The Lagrangian of the system.

(b) The generalized momenta and corresponding Hamiltonian.

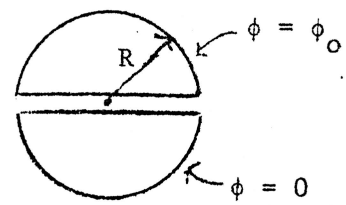
(c) The equation of motion for the coordinate r as a function of time.

(d) If $d\phi/dt = 0$, show that the particle can execute small oscillations about the lowest point of the paraboloid, and find the frequency of these oscillations.

Part II - Electricity and Magnetism

Answer 3 out of 5 questions (6-10)

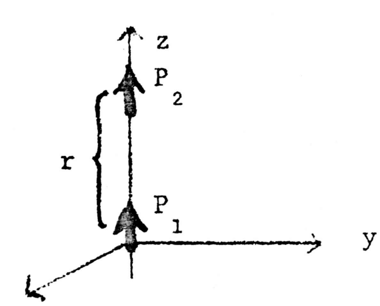
6. A conducting spherical shell of radius R is cut in half. The two hemispherical pieces are electrically separated from each other but are left close together as shown in the sketch so that the distance separating the two halves can be neglected. The upper half is maintained at a potential $\phi = \phi_o$ and the lower half is maintained at a potential $\phi = 0$.



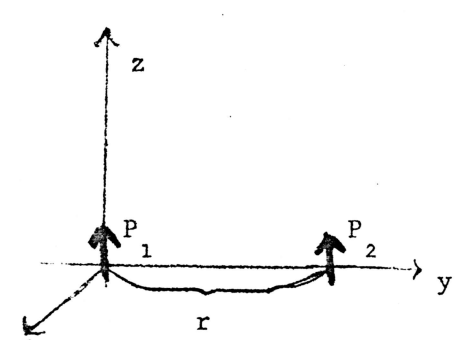
Calculate the electrostatic potential ϕ at all points in space outside of the surface of the conductors. Neglect terms falling faster than $1/r^4$ (i.e. keep terms up to and including those with $1/r^4$ dependence), where r is the distance from the center of the conductor. Hints: Start with the solution of Laplace's equation in the appropriate coordinate system. The boundary condition of the surface of the conductor will have to be expanded in a series of Legendre polynomials.

$$P_o(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \tfrac{3}{2}x^2 - \tfrac{1}{2}$$
$$P_3(x) = \tfrac{5}{2}x^3 - \tfrac{3}{2}x$$

7. An electric dipole with dipole moment $\vec{p}_1 = p_1 \hat{k}$ is located at the origin of the coordinate system. A second dipole of dipole moment $\vec{p}_2 = p_2 \hat{k}$ is located at a) on the +z axis a distance r from the origin or b) on the +y axis a distance r from the origin. Show that the force between the two dipoles is attractive in case a, and repulsive in case b. Calculate the magnitude of the force in the two cases.



(a)



(b)

8. Some isotropic dielectrics become birefringent (doubly refracting) when they are placed in a static external magnetic field. Such magnetically-biased materials are said to be gyrotropic and are characterized by a permittivity ε and a constant "gyration vector" $\vec{g}$. In general, $\vec{g}$ is proportional to the static magnetic field which is applied to the dielectric. Consider a monochromatic plane wave

$$\begin{Bmatrix} \vec{E}(\vec{x},t) \\ \vec{B}(\vec{x},t) \end{Bmatrix} = \begin{Bmatrix} \vec{E}_o \\ \vec{B}_o \end{Bmatrix} e^{i(k\hat{n} \cdot \vec{x} - \omega t)}$$

traveling through a gyrotropic material. ω is the given angular frequency of the wave, and $\hat{n}$ is the given direction of propagation. $\vec{E}_o$, $\vec{B}_o$, and k are constants to be determined. For a non-conducting ($\sigma = 0$) and non-permeable ($\mu = 1$) gyrotropic material, the electric displacement $\vec{D}$ and the electric field $\vec{E}$ are related by

$$\vec{D} = \varepsilon\vec{E} + i(\vec{E} \times \vec{g})$$

where the permittivity ε is a positive real number and where the "gyration vector" $\vec{g}$ is a constant real vector. Consider plane waves which propagate in the direction of $\vec{g}$, with $\vec{g}$ pointing along the z-axis:

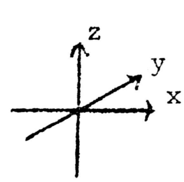
$$\vec{g} = g\hat{z} \text{ and } \hat{n} = \hat{z}.$$

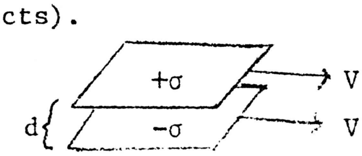
a) Starting from Maxwell's equations, find the possible values for the index of refraction $N \equiv kc/\omega$. Express your answers in terms of the constants ε and g.

b) For each possible value of N, find the corresponding polarization $\vec{E}_o$.

9. Two large flat conducting plates are separated by a distance D and are connected together by a wire. A point charge with total charge Q is placed midway between the two plates (above and below their respective centers). Find an expression for the surface charge induced on the lower plate as a function of D, Q and x (the distance from the center of the plate).

10. Two large parallel plates (non-conducting), separated by a distance d and oriented as shown, move together along the x-axis with velocity v, not necessarily small compared to c. The upper and lower plates have uniform surface charge density $+\sigma$ and $-\sigma$, respectively, in the rest frame of the plates. Find the magnitude and direction of the electric and magnetic fields between the plates (neglecting edge effects).





Thursday, January 3, 1985

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DEPARTMENT OF PHYSICS

Ph.D. Qualifying Examination

MODERN PHYSICS

ANSWER EACH QUESTION IN A SEPARATE BOOK AND MARK THE QUESTION NUMBER AND YOUR NAME CLEARLY ON EACH BOOK.

ANSWER 6 OUT OF 9 QUESTIONS.

Useful Constants

		Uncert. (ppm)
N_A	$= 6.022\ 045(31)\times10^{23}$ mole^{-1}	5.1
V_m	$= 22413.83(70)$ cm^3 mole^{-1} = molar volume of ideal gas at STP	31
c	$= 2.997\ 924\ 58(1.2)\times10^{10}$ cm sec^{-1}	0.004
e	$= 4.803\ 242(14)\times10^{-10}$ esu $= 1.602\ 189\ 2(46)\times10^{-19}$ coulomb	2.9; 2.9
1 MeV	$= 1.602\ 189\ 2(46)\times10^{-6}$ erg	2.9
$\hbar=h/2\pi$	$= 6.582\ 173(17)\times10^{-22}$ MeV sec $= 1.054\ 588\ 7(57)\times10^{-27}$ erg sec	2.6; 5.4
$\hbar c$	$= 1.973\ 285\ 8(51)\times10^{-11}$ MeV cm $= 197.32858(51)$ MeV fermi	2.6; 2.6
$(\hbar c)^2$	$= 0.389\ 385\ 7(20)$ GeV^2 mb	5.2
α	$= e^2/\hbar c = 1/137.03604(11)$	0.82
$k_{\text{Boltzmann}}$	$= 1.380\ 662(44)\times10^{-16}$ erg °K^{-1}	32
	$= 8.61735(28)\times10^{-11}$ MeV °K^{-1} = 1 eV/11604.50(36) °K	32; 31
$\sigma_{\text{Stef. Boltz.}}$	$= 5.67032(71)\times10^{-5}$ erg sec^{-1} cm^{-2} °K^{-4}	125
	$= 3.53911(44)\times10^{7}$ eV sec^{-1} cm^{-2} °K^{-4}	125
m_e	$= 0.511\ 003\ 4(14)$ MeV $= 9.109\ 534(47)\times10^{-28}$ g	2.8; 5.1
m_p	$= 938.2796(27)$ MeV $= 1836.15152(70)\ m_e = 6.722\ 775(39)\ m_{\pi^\pm}$	2.8; 0.38; 5.8
	$= 1.007\ 276\ 470(11)$ amu	0.011
1 amu	$= 1/12\ m_{C^{12}} = 931.5016(26)$ MeV	2.8
m_d	$= 1875.6280(53)$ MeV	2.8
r_e	$= e^2/m_ec^2 = 2.817\ 938\ 0(70)$ fermi (1 fermi $= 10^{-13}$ cm)	2.5
$\lambda\!\!\!^-_e$	$= \hbar/m_ec = r_e\alpha^{-1} = 3.861\ 590\ 5(64)\times10^{-11}$ cm	1.6
$a_{\infty\text{Bohr}}$	$= \hbar^2/m_ee^2 = r_e\alpha^{-2} = 0.529\ 177\ 06(44)$ Å (1 Å $= 10^{-8}$ cm)	0.82
σ_{Thomson}	$= (8/3)\pi r_e^2 = 0.665\ 244\ 8(33)$ barn (1 barn $= 10^{-24}$ cm^2)	4.9
μ_{Bohr}	$= e\hbar/2m_ec = 0.578\ 837\ 85(95)\times10^{-14}$ MeV gauss^{-1}	1.6
μ_N	$= e\hbar/2m_pc = 3.152\ 451\ 5(53)\times10^{-18}$ MeV gauss^{-1}	1.7
μ_p/μ_{Bohr}	$= 0.001\ 521\ 032\ 209(16)$	0.011
$1/2\,\omega^e_{\text{cyclotron}}$	$= e/2m_ec = 8.794\ 024(25)\times10^{6}$ rad sec^{-1} gauss^{-1}	2.8
$1/2\,\omega^p_{\text{cyclotron}}$	$= e/2m_pc = 4.789\ 378(14)\times10^{3}$ rad sec^{-1} gauss^{-1}	2.8

Hydrogen-like atom (nonrelativistic, μ = reduced mass):

$$\left.\frac{v}{c}\right)_{rms} = \frac{z\alpha}{n};\quad E_n = \frac{\mu}{2}v^2 = \frac{\mu}{2}\left(\frac{cz\alpha}{n}\right)^2;\quad a_n = \frac{n^2\hbar}{\mu zc\alpha}$$

$R_\infty = m_ee^4/2\hbar^2 = m_ec^2\alpha^2/2$	$= 13.605\ 804(36)$ eV (Rydberg)	2.6
	$= m_ec\alpha^2/2h = 109\ 737.3177(83)$ cm^{-1}	0.075
$pc = 0.3\ H\rho$ (MeV, kilogauss, cm)		
1 year (sidereal)	= 365.256 days $= 3.1558\times10^7$ sec ($\approx\pi\times10^7$ sec)	
density of dry air	= 1.204 mg cm^{-3} (at 20°C, 760 mm)	
acceleration by gravity	= 980.62 cm sec^{-2} (sea level, 45°)	
gravitational constant	$= 6.6720(41)\times10^{-8}$ cm^3 g^{-1} sec^{-2}	615
1 calorie (thermochemical)	= 4.184 joules	
1 atmosphere	= 1.01325 bar (1 bar $= 10^6$ dynes cm^{-2})	
1 eV per particle	= 11604.50(36) °K (from $E = kT$)	31

NUMERICAL CONSTANTS

π	= 3.141 592 7	1 rad	= 57.295 779 5 deg	$\sqrt{\pi}$	= 1.772 453 85
e	= 2.718 281 8	$1/e$	= 0.367 879 4	$\sqrt{2}$	= 1.414 213 6
ln2	= 0.693 147 2	ln10	= 2.302 585 1	$\sqrt{3}$	= 1.732 050 8
$\log_{10}2$	= 0.301 030 0	$\log_{10}e$	= 0.434 294 5	$\sqrt{10}$	= 3.162 277 7

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CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

	J J ... / M M ...
m_1 m_2 / m_1 m_2 / . .	Coefficients

1/2 × 1/2

m_1	m_2		
		$J=1$, $M=+1$	
+1/2	+1/2	1	
		$J=1$, $M=0$	$J=0$, $M=0$
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2
		$J=1$, $M=-1$	
-1/2	-1/2	1	

1 × 1/2

m_1	m_2		
		3/2, +3/2	
+1	+1/2	1	
		3/2, +1/2	1/2, +1/2
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3
		3/2, -1/2	1/2, -1/2
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3
		3/2, -3/2	
-1	-1/2	1	

2 × 1/2

m_1	m_2		
		5/2, +5/2	
+2	+1/2	1	
		5/2, +3/2	3/2, +3/2
+2	-1/2	1/5	4/5
+1	+1/2	4/5	-1/5
		5/2, +1/2	3/2, +1/2
+1	-1/2	2/5	3/5
0	+1/2	3/5	-2/5
		5/2, -1/2	3/2, -1/2
0	-1/2	3/5	2/5
-1	+1/2	2/5	-3/5
		5/2, -3/2	3/2, -3/2
-1	-1/2	4/5	1/5
-2	+1/2	1/5	-4/5
		5/2, -5/2	
-2	-1/2	1	

3/2 × 1/2

m_1	m_2		
		2, +2	
+3/2	+1/2	1	
		2, +1	1, +1
+3/2	-1/2	1/4	3/4
+1/2	+1/2	3/4	-1/4
		2, 0	1, 0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2
		2, -1	1, -1
-1/2	-1/2	3/4	1/4
-3/2	+1/2	1/4	-3/4
		2, -2	
-3/2	-1/2	1	

2 × 1

m_1	m_2			
		3, +3		
+2	+1	1		
		3, +2	2, +2	
+2	0	1/3	2/3	
+1	+1	2/3	-1/3	
		3, +1	2, +1	1, +1
+2	-1	1/15	1/3	3/5
+1	0	8/15	1/6	-3/10
0	+1	6/15	-1/2	1/10
		3, 0	2, 0	1, 0
+1	-1	1/5	1/2	3/10
0	0	3/5	0	-2/5
-1	+1	1/5	-1/2	3/10
		3, -1	2, -1	1, -1
0	-1	6/15	1/2	1/10
-1	0	8/15	-1/6	-3/10
-2	+1	1/15	-1/3	3/5
		3, -2	2, -2	
-1	-1	2/3	1/3	
-2	0	1/3	-2/3	
		3, -3		
-2	-1	1		

3/2 × 1

m_1	m_2			
		5/2, +5/2		
+3/2	+1	1		
		5/2, +3/2	3/2, +3/2	
+3/2	0	2/5	3/5	
+1/2	+1	3/5	-2/5	
		5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2	-1	1/10	2/5	1/2
+1/2	0	3/5	1/15	-1/3
-1/2	+1	3/10	-8/15	1/6
		5/2, -1/2	3/2, -1/2	1/2, -1/2
+1/2	-1	3/10	8/15	1/6
-1/2	0	3/5	-1/15	-1/3
-3/2	+1	1/10	-2/5	1/2
		5/2, -3/2	3/2, -3/2	
-1/2	-1	3/5	2/5	
-3/2	0	2/5	-3/5	
		5/2, -5/2		
-3/2	-1	1		

1 × 1

m_1	m_2			
		2, +2		
+1	+1	1		
		2, +1	1, +1	
+1	0	1/2	1/2	
0	+1	1/2	-1/2	
		2, 0	1, 0	0, 0
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3
		2, -1	1, -1	
0	-1	1/2	1/2	
-1	0	1/2	-1/2	
		2, -2		
-1	-1	1		

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

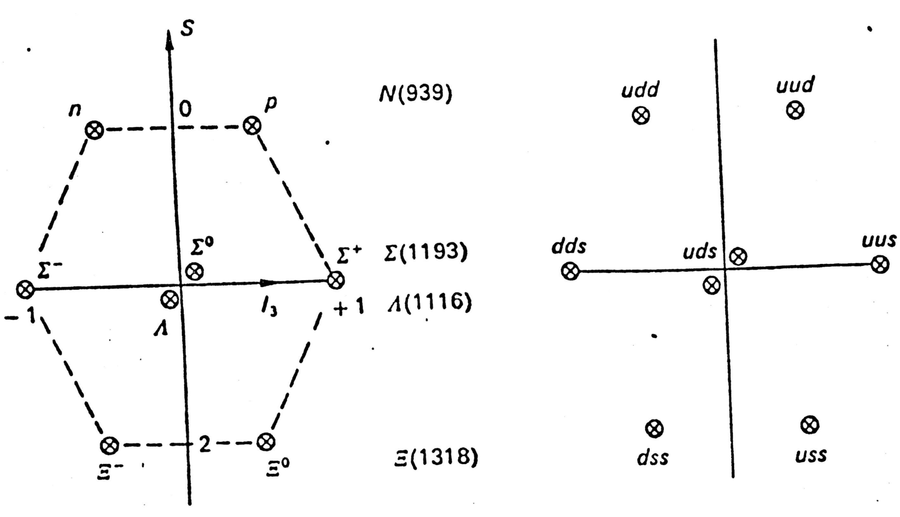

Fig. 5.4 The baryon octet of spin-parity $\frac{1}{2}^+$. The observed states are given on the left, and quark flavor assignments on the right.

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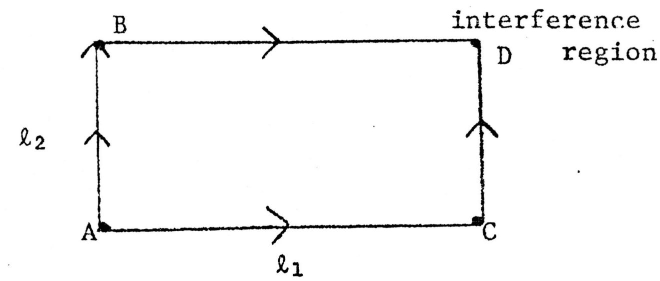
1. A monoenergetic beam selected from thermal neutrons is split and then brought together again as shown in the figure $AC = BD = \ell_1$, $AB = CD = \ell_2$

Originally, the plane ABCD is horizontal. It is then rotated by 90° around AC so that AB and CD are vertical. In the course of this rotation the intensity at D is found to go through a series of maxima and minima. Explain this phenomenon and determine how many maxima will appear as the ABCD is rotated from horizontal to vertical.



2. Show that for a non-relativistic one dimensional system with a smooth finite potential which depends only on the spatial coordinate, the ground state wave function always has constant phase.

3. A hydrogen atom is in a state given by .

$$|n,j,m,\ell,s\rangle = |3,\tfrac{5}{2},-\tfrac{3}{2},2,\tfrac{1}{2}\rangle$$

$$= \frac{1}{\sqrt{5}}\,|R_{32}(r)\rangle\left\{|2,-2\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + 2|2,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right\}$$

(a) Write down the values obtained for measurements of

(1) J^2

(2) J_z

(3) L^2

(4) L_z

(5) S^2

(6) S_z

(7) $\vec{L}\cdot\vec{S}$

(8) Energy (in completely non-relativistic approximation.)

Give your answers in terms of fundamental constants. Where more than one value is possible, give the relative probability of obtaining each value and calculate the expectation value.

[part (b) is on the next page!]

(b) In the decay of this state - what spontaneous transitions are permitted by the electric dipole selection rules? What, in order or magnitude, are the relative transition probabilities?

4. Discuss qualitatively the shift due to a constant external electric field E_0 of the n = 2 energy levels of hydrogen. Neglect spin, but include the observed zero field splitting of the 2s and 2p states W,

$$W \equiv E_{2s} - E_{2p} \sim 10^{-5} \text{ eV}.$$

Consider separately the cases $E_0 a_0 >> W$ and $E_0 a_0 << W$, where a_0 is the Bohr radius.

5.

(a) Using the fact that electrons in a molecule are confined to a volume typical of a molecule estimate the spacing in energy of the excited states of the electrons. ($E_{electronic}$)

(b) As nuclei in a molecule move they distort electronic wave functions. This distortion changes the electronic energy. The nuclei oscillate about positions of minimum total energy: electrons plus the repulsive coulomb energy between nuclei. Estimate the frequency and therefore the energy of these vibrations ($E_{vibration}$) by saying that a nucleus is in a harmonic oscillator potential.

(c) Estimate the deviations from the equilibrium sites of the nuclei.

(d) Estimate the energy of the rotational excitations (E_{rot})

(e) Estimate the ratio of $E_{elect} : E_{vib} : E_{rot}$ in terms of the ratio of electron mass to nuclear mass $\frac{M_e}{M_n}$.

6. Consider the hyperon non-leptonic weak decays

$$\Lambda^0 \to p\pi^-$$
$$\Lambda^0 \to n\pi^0$$
$$\Sigma^- \to n\pi^-$$
$$\Sigma^+ \to p\pi^0$$
$$\Sigma^+ \to n\pi^+$$
$$\Xi^- \to \Lambda^0\pi^-$$
$$\Xi^0 \to \Lambda^0\pi^0$$

6. con't.

On assuming that these $(\Delta S) = 1$ weak decays satisfy the $\Delta I = \frac{1}{2}$ rule, use the attached tables to find the values of x,y and z below:

$$x = \frac{A(\Sigma^0 \to p\pi^-)}{A(\Lambda^0 \to n\pi^0)}$$

$$y = \frac{A(\Sigma^+ \to \pi^+ n) - A(\Sigma^- \to \pi^- n}{A(\Sigma^+ \to \pi^0 p)}$$

$$z = \frac{A(\Xi^0 \to \Lambda^0 \pi^0)}{A(\Xi^- \to \Lambda^0 \pi^-)}$$

A denotes the transition amplitude.

7. The ρ - meson is a meson resonance with mass of 769 MeV and width of 154 MeV. It can be produced experimentally by bombarding a hydrogen target with a π - meson beam,

$$\pi^- p \to \rho^0 n$$

(a) What is the lifetime and mean decay distance for a 5 GeV ρ^0?

(b) What is the π^- threshold energy for producing ρ^0 mesons?

(c) If the production cross section is 1 mb = 10^{-27} cm^2 and the liquid hydrogen target is 30 cm long, how many ρ^0 mesons are produced on the average per incident π^-? (The density of liquid hydrogen is .07 gr/c.c.)

(d) ρ^0 mesons decay almost instantaneously into $\pi^+ + \pi^-$. Given that the ρ^0 is produced in the forward direction in the lab frame with an energy of 5 GeV, what is the minimum opening angle between the outgoing π^+ and π^- in the lab frame?

8. Two accelerator facilities are under construction which will produce the neutral intermediate vector boson, Z^0, via the process

$$e^- + e^+ \to Z^0$$

The mass of the Z^0 is $M_{Z^0} = 92$ GeV/c^2

(a) Find the energy of the electron beams needed for the colliding beam facility under construction.

Assume that a fixed target facility is to be built, such that a beam of e^+ will strike a target of e^- at rest.

(b) What is the required e^+ beam energy for this case?

(c) What is the energy and velocity of the Z^0 (in the lab) after production?

[question continues on the next page]

8. con't.

(d) Find the maximum energy in the lab frame of muons from the subsequent decay $Z^0 \rightarrow \mu^+ + \mu^-$

9. It is suggested that non-rotating black holes can radiate all possible particles with equal a-priori probability. Assume that all black holes of given mass M radiate identically, estimate by dimensional analysis only, the following:

(a) The classical radius of the black hole of mass M.

(b) Assuming the black hole radiates as an antenna what is the energy of a typical radiated particle? Is there an upper limit on such energy?

(c) Typical time between emission of two particles and the energy emitted per unit time.

(d) The mass M(t) of the black hole as a function of time and the hole's lifetime.

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Friday, January 4, 1985

DEPARTMENT OF PHYSICS
Ph.D. Qualifying Examination
GENERAL PHYSICS

Answer each question in a separate book and mark the question number and your name clearly on each book.

Answer eight out of the following ten questions.

1. Many of the fundamental discoveries of the 20th centry in basic physics have led to a wide variety of practical applications. Examples of such are:

    a) Nuclear Magnetic Resonance, used in the determination of chemical structure or in medicine.

    b) Hydrogen Maser, used as an atomic clock to provide timing for more accurate navigation.

    c) An IR or visible Laser, used in surgery or in machining metals.

    d) The electron synchroton, used to provide radiation for lithography in the fabrication of large scale integrated circuits.

    e) The strong interaction of pions, used for radiation therapy in the treatment of cancer.

    f) The Fermi levels of doped semiconductors, used in the production of transistors.

    For two of the above:

    I. Give a quantitative description of the fundamental discovery indicating the details of the experiments involved and explaining how they fit into the theoretical background.

    II. Explain in detail how the applications are carried out, emphasizing the features which make them superior to previous techniques.

2. Tritium, the isotope H^3, undergoes beta-decay with a half-life of 12.5 years. An enriched sample of hydrogen gas, containing 0.1 gram of tritium, produces 21 calories of heat per hour.

    a) For these data, calculate the average energy of the β-particles emitted.

    b) What specific measurements on the beta spectrum (including the decay nucleus) indicate that there is an additional decay product and specifically that it is light and neutral.

    c) Give a critical, quantitative analysis of how a careful measurement of the beta spectrum of tritium can be used to determine (or put an upper limit on) the mass of the electron's neutrino.

3. Make a numerical estimate of 4 of the following 6 quantities (accurate to within an order of magnitude). Explain thoroughly the reasoning used in obtaining your result.

   a) The elastic modulus of steel (in dynes/cm^2).

   b) The height of tides on the ocean.

   c) The viscosity (in dynes/cm^2) for H_2 at room temperature and one atmosphere pressure.

   d) The speed of terrestrial winds. (You may use the solar constant 2 cal/cm^2/min.).

   e) The average magnetic field in a permanent magnet.

   f) The maximum height of mountains on Earth. (Assume that mountains break down under the stress produced by the gradient of gravity when the strain exceeds 10^{-3}).

Your estimate should be made using fundamental atomic or astronomical constants.

4. The expression for the intensity of radiation emitted per unit frequency per unit solid angle by a particle of charge e travelling with uniform velocity $\beta = v/c$ through a medium of index of refraction n is

$$\frac{dI(\omega,\theta)}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\, n\, \sin^2\theta \left| \int_{-T}^{T} e^{i\omega t(1-n\,\beta\cos\theta)}\, dt \right|^2$$

where 2T is the time spent in the medium and θ is the direction of the radiation with respect to the direction of motion of the particle. For the familiar ideal case of Cerenkov radiation $T \to \infty$ and the above integral becomes a δ-function. All radiation is emitted in the direction $\theta = \theta_c$ and the intensity is proportional to the distance traversed $L = 2vT$.

   a) Find θ_c and explain the origin of the exponential in the integral.

   b) Do the integral in the given equation for the more general case of T finite.

   c) Take the case $n = 1$ and θ small ($\sin\theta \approx \theta$, $\cos\theta \approx 1$) to find an expression for the angular distribution of the forward radiation emitted by a charged particle traversing vacuum over the finite distance $L = 2vT$.

   d) Explain, referring to the expression in (c), how radiation can be emitted by a charged particle travelling at uniform velocity in a vacuum inside a metal vessel.

5. In a recently televised program on whales, it was reported that, "Scientists have discovered that whales are quite near-sighted out of water,"[that is, when both the whales and the objects they are looking at are out of water]. Assuming that whales have good eyesight <u>in</u> water:

   a) Show, with the help of a sketch, that whales must be near-sighted out of water.

   b) Suppose you are asked to design a pair of eyeglasses (<u>not</u> contact lenses) for a whale, so that it can see out of water. Should the lenses be converging or diverging? If the radius of curvature of the outer surface of the lens of the whale's eye is r and the glasses are to sit a distance $d < r$ from this surface, find the focal length of the lens of the glasses. You may make the usual small angle approximations.

6. A Carnot engine operates on one mole of ideal classical monatomic gas. At the beginning of the isothermal expansion the temperature is $4T_o$ and the volume is V_o. At the beginning of the isothermal compression the temperature is T_o and the volume is $64V_o$. Let W denote the work done by the gas per cycle. Now suppose a similar engine operates on one mole of classical diatomic gas, with all parameters the same as before, except that the work per cycle is W'. Find W'/W.

7. Devise a method to measure the bandwidth, $\frac{\Delta\omega}{\omega}$, of a quasimonchromatic incoherent light source. Explain the method and derive the bandwidth range to which it is applicable.

8. The Stanford Linear Accelerator has a length of 2 miles (= 3.22 km) and accelerates electrons to a final energy of 20 GeV (= 2×10^{10} eV).

   a) Assuming that the accelerating electric field is uniform and that the initial kinetic energy of the electrons is zero, calculate the length of the accelerator as seen by the electrons in the beam (M_e = .511 MeV/c^2).

   b) If the 20 GeV electron beam collides with a stationary proton target, calculate the energy in the center of mass of the electron-proton system (M_p = 938 MeV/c^2).

9. The following ratios of decay rates have the approximate values shown. Give a short explanation of why they have these values. (Answer six out of seven.)

	RATIO	VALUE
a)	$\dfrac{\Gamma(\pi^o \rightarrow \gamma + e^+ + e^-)}{\Gamma(\pi^o \rightarrow \gamma + \gamma)}$	.01
b)	$\dfrac{\Gamma(K^o_S \rightarrow \pi^o + \pi^o)}{\Gamma(K^o_S \rightarrow \pi^+ + \pi^-)}$	½

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9. (continued)

	RATIO	VALUE
c)	$\dfrac{\Gamma(\mu^+ \to e^+ + \gamma)}{\Gamma(\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu)}$	0
d)	$\dfrac{\Gamma(\Lambda^o \to p + \pi^-)}{\Gamma(\Lambda^o \to n + \pi^o)}$	2
e)	$\dfrac{\Gamma(Z^o \to e^+ + e^-)}{\Gamma(Z^o \to \mu^+ + \mu^-)}$	1
f)	$\dfrac{\Gamma(K_L^o \to \pi^+ + \pi^-)}{\Gamma(K_S^o \to \pi^+ + \pi^-)}$	2×10^{-3}
g)	$\dfrac{\Gamma(K^+ \to \pi^+ + \gamma)}{\Gamma(K^+ \to \pi^+ + \pi^o)}$	0

10. a) Given that the mass of the sun is 2×10^{33}g, estimate the number of electrons in the sun. Assume the sun is largely composed of atomic Hydrogen.

b) In a white dwarf star of one solar mass the atoms are all ionized and contained in a sphere of radius 2×10^9 cm. Find the Fermi Energy of the electrons in eV.

c) If the temperature of the white dwarf is 10^7K, discuss whether the electrons and/or nucleons in the star are degenerate.

d) If the above number of electrons were contained in a pulsar of one solar mass and of radius 10km, find the order of magnitude of their Fermi Energy.

Thursday, January 2, 1986

COLOMBIA

DEPARTMENT OF PHYSICS

Ph.D. Qualifying Examination

MODERN PHYSICS

Closed Book - Four Hours

ANSWER EACH QUESTION IN A SEPARATE BOOK AND MARK THE QUESTION NUMBER AND THE ALPHABETICAL IDENTIFICATION CODE THAT WAS ASSIGNED TO YOU CLEARLY ON EACH BOOK.

Answer 5 out of 6 questions.

1. Use the Bohr-Sommerfeld quantization rule to calculate the allowed energy levels of a ball which is bouncing elastically in the vertical direction.

2. One dimensional particles of total energy $E > 0$ encounter a one dimensional potential well of depth V_o and width L. Calculate the transmission coefficients:
   a) present the general equation it satisfies.
   b) solve for $E << V_o$.
   c) solve for $E >> V_o$.
   d) Is there a situation where the transmission of particles is perfect? (i.e. transparent).
   e) Can you see any application of this phenomenon?

3. A fact: For a one dimensional Schrödinger equation

$$H_1 = \frac{1}{2m} p^2 + V(x) \qquad [x,p] = i$$

with $V(x) \leqslant 0$ for all x and $\int V(x)dx < 0$, there always exists a negative energy state (= bound state).

The Actual Question

Consider now a three dimensional Schrödinger equation with

$$H_3 = \frac{1}{2m} \vec{p}^2 + V(\vec{x}) \qquad [x_i,p_j] = i\delta_{ij}$$

and again $V(\vec{x}) \leqslant 0$ for all $\vec{x}$ and $\int V(\vec{x})\, d^3x < 0$. Does H_3 still always have a bound state for all such $V(\vec{x})$?

Either
   a) Find a counterexample.
   b) Prove that H_3 always has a bound state.

4. Two identical particles of spin $\frac{1}{2}$ are confined to a cubical box whose sides are $d = 10^{-8}$cm in length. There is an attractive potential between pairs of particles of strength $V_0 = 10^{-3}$ eV, acting whenever the distance between the particles is less than 10^{-10} cm. Using non-relativistic perturbation theory, calculate the ground state energy. Take the mass of the particles to be that of the electron.

5. Coherent states of one dimensional harmonic oscillator are defined as eigenvectors of the annihilation operator a:

$$a|\Psi_\lambda> = \lambda|\Psi_\lambda>, \ \lambda \text{ a complex number.}$$

a) Find an expression for $|\Psi_\lambda>$ as a sum over harmonic oscillator eigenstates $|n>$.

b) Show that the probabilities of the various states in this sum are given by a Poisson distribution:
i.e. $P(n) = e^{-p} \, p^n/_{n!}$, and find p.

6. Consider the one dimensional quantum mechanical system given by the Hamiltonian

$$H = \tfrac{1}{2}p^2 + \frac{\lambda^2}{2x^2} + \tfrac{1}{2}x^2 \qquad \text{where } [x,p] = i \quad \lambda > 0$$

In analogy with the construction of the eigenvalues of the angular momentum operator L_3, we propose to find the eigenvalues of H, with the help of raising and lowering operators.

a) Show that the operators H, K and D have simple commutation relations with each other, where

$$K = \tfrac{1}{2}p^2 + \frac{\lambda^2}{2x^2} - \tfrac{1}{2}x^2$$

$$D = \frac{i}{2}(xp + px) = ixp + \tfrac{1}{2}$$

b) Show that $C_o = H^2 + D^2 - K^2$ commutes with H, D and K using the commutation relations derived in a). (C_o is analogous to $\vec{L}^2$; H, D and K are analogous to L_1, L_2 and L_3).

c) Calculate C_o explicitly in terms of x and p, using the definitions of H, D and K. Check your result of b).

d) Show that D + K, applied to an eigenstate of H reproduces an eigenstate of H, with <u>higher</u> eigenvalue. This is the raising operator. Write down the lowering operator.

e) Find all eigenvalues of H.

Friday, January 3, 1986

COLUMBIA

DEPARTMENT OF PHYSICS

Ph.D. Qualifying Examination

GENERAL PHYSICS

Closed Book - 4 Hours

ANSWER EACH QUESTION IN A SEPARATE BOOK. MARK THE QUESTION NUMBER AND THE ALPHABETICAL IDENTIFICATION CODE THAT WAS ASSIGNED TO YOU CLEARLY ON EACH BOOK.

Answer FIVE of the following six questions:

1. A closed cylinder is divided into three volumes separated by walls. Each part contains 1 mole of an ideal gas, but the three gases are of different kinds. The temperatures are equal. One subsequently removes the walls separating the gases. Calculate the change in entropy (1) thermodynamically; (2) statistically.

2. Protons have a nuclear gyromagnetic ratio corresponding to 42.6 MHz/10kG.

   (a) At room temperature and in a constant magnetic field of 10kG, what fraction of the protons in water are in their lowest spin state?

   (b) A 1G linearly polarized magnetic field at 42.6MHz is applied to this sample for 10 μsec along an axis normal to the 10kG field. Using perturbation theory, calculate the probability that a proton initially in the ground spin state will be promoted to the excited spin state. Estimate the error in your calculation.

   (c) What would the error be if the rf field's amplitude is raised to 10G?

3. Using the following constants and any other information that you know, estimate:

   (a) The temperature on the surface of the sun.

   (b) The solar radius.

   (c) The total energy output of the sun.

   (d) The solar energy reaching the earth.

Constants:

Planck's $h = 6.6 \times 10^{-27}$ erg sec

Stefan-Boltzmann's $\sigma = \pi^2 k^4 / 60\hbar^3 c^2 = 5.7 \times 10^{-5}$ erg sec^{-1} cm^{-2} K^{-4}

Radius of earth $r = 6.4 \times 10^6$ m

Radius of earth's orbit $R_0 = 1.5 \times 10^{11}$m

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4. For each of two of the following properties, describe carefully an experiment in which a precise measurement can be made. In each case, indicate in detail the equipment to be used, the quantities to be measured, how the measurements are to be combined to give the desired result and the accuracy which can be achieved.

(a) The binding energy of the deuteron.

(b) The lifetime of the neutral pion.

(c) The lifetime of the free neutron.

(d) The energy of the K-alpha x-ray of lead.

(e) The mass (or limit of mass) of the neutrino.

(f) The magnetic moment (or g-2) of the electron.

5. Suppose that one detects a flux of very energetic particles from the binary star system Cygnus X-3, with the particles always arriving within a fixed 15 minute portion of the 4.8 hour binary period of Cygnus X-3. Suppose also that the particle energies vary by at most a factor of 2 and that the maximum energy is 10^6GeV. Take the distance from Cygnus X-3 to earth to be 40,000 light-years.

(a) Obtain an upper bound on the mass of the particle.

(b) If that mass is at least 1GeV/c^2, what is the shortest possible half-life consistent with the data?

6. Each of the following branching ratios is determined by a selection rule or conservation law. In six of the eight following cases explain the principle involved and, if possible, estimate the ratio.

(a) $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}$

(b) $\dfrac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}$

(c) $\dfrac{\Gamma(K^+ \to \pi^+ + e^+ + e^-)}{\Gamma(K^+ \to \pi^0 + e^+ + \nu_e)}$

(d) $\dfrac{\Gamma(\rho \to \pi^+ \pi^- \pi^0)}{\Gamma(\rho \to \pi^+ \pi^-)}$

(e) $\dfrac{\Gamma(N^{*+} \to \pi^0 p)}{\Gamma(N^{*++} \to \pi^+ p)}$

(f) $\dfrac{\Gamma(\mu^- \to e^- + \gamma)}{\Gamma(\mu^- \to e^- + \nu_\mu + \bar{\nu}_e)}$

(g) $\dfrac{\Gamma(\pi^0 \to e^+ e^-)}{\Gamma(\pi^+ \to e^+ \pi^0 \nu_e)}$

(h) $\dfrac{\Gamma(p \to e^+ \pi^0)}{\Gamma(\pi^+ \to e^+ \pi^0 \nu_e)}$

Monday, January 6, 1986

COLUMBIA

DEPARTMENT OF PHYSICS

Ph.D. Qualifying Examination

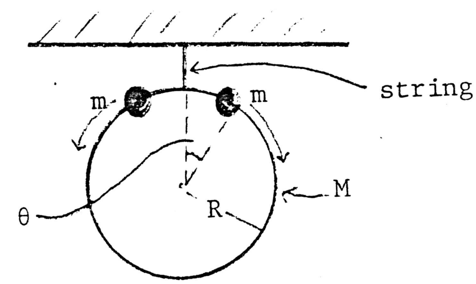
CLASSICAL PHYSICS

Closed Book - Four Hours

ANSWER EACH QUESTION IN A SEPARATE BOOK AND MARK THE QUESTION NUMBER AND THE ALPHABETICAL IDENTIFICATION CODE THAT WAS ASSIGNED TO YOU CLEARLY ON EACH BOOK.

Part I - MECHANICS

Answer 3 out of 4 questions (1-4)



1. Two beads, each of mass m, slide on a massive (M) hoop which is suspended from a light string. Assume the beads slide without friction and are released from rest at the top of the hoop.
   a) If M = 0, at what angle θ will the hoop begin to rise?
   b) For an arbitrary mass hoop, what is the minimum mass of a bead that is required for the hoop to rise? Express your answer in terms of M.

2. According to Yukawa's theory of nuclear forces, the attractive force between a neutron and a proton has the potential

$$V(r) = \frac{K\, e^{-\alpha r}}{r} \qquad K < 0$$

   a) Find ℓ (the angular momentum) and E (the total energy) for motion in a circle of radius a.
   b) Find the period of circular motion and the period of small radial oscillation.
   c) Show that the nearly circular orbits are almost closed when αa is very small.

3. The motion of a compressible and viscous fluid is described by the Navier-Stokes equation

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\nabla\vec{P} + \alpha\nabla^2\vec{v} + \beta\vec{\nabla}\,(\mathrm{div}\,\vec{v})$$

and the continuity equation

$$\frac{\partial\rho}{\partial t} + \mathrm{div}\,(\rho\vec{v}) = 0.$$

$\vec{v} = \vec{v}(\vec{x},t)$ is the fluid's velocity and $\rho = \rho(\vec{x},t)$ is the fluid's mass density. The hydrostatic pressure $P = P(\rho)$ depends on the mass density ρ (as well as on the temperature). The viscosity coefficients α and β are positive constants.

(continued on page 2)

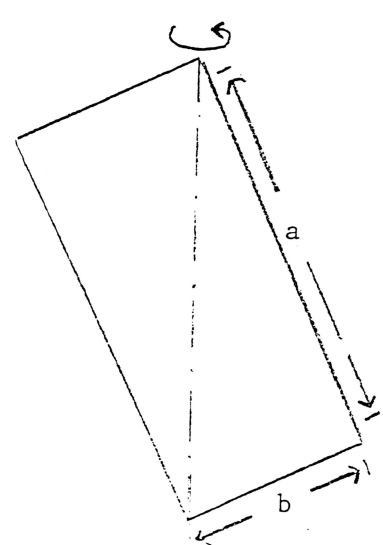
The effects of gravity are assumed to be negligible. The equilibrium state of the fluid has $\vec{v}(\vec{x},t)$ equal to zero and $\rho(\vec{x},t)$ equal to a constant ρ_e. Consider small-amplitude departures from these equilibrium values (linearized equations) as a monochromatic plane wave of angular frequency ω travels through the fluid in the +z direction:

$$\rho(\vec{x},t) = \rho_e + (\delta\rho_o)e^{i(kz - \omega t)}$$
$$\vec{v}(\vec{x},t) = 0 + (\delta\vec{v}_o)e^{i(kz - \omega t)}$$
$$P\left(\rho(\vec{x},t)\right) = P(\rho_e) + \left(\frac{\partial P}{\partial \rho}\right)_{\rho_e}(\delta\rho_o)e^{i(kz - \omega t)}$$

Find the possible values of the propagation constant k and the corresponding polarizations $\delta\vec{v}_o$ (transverse and longitudinal). Express your answers in terms of ω, ρ_e, α, β, and the constant C defined as

$$C = \left(\frac{\partial P}{\partial \rho}\right)_{\rho_e}$$

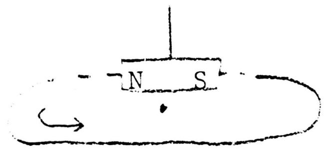
4. A thin rectangular plate with uniform density is mounted so that its diagonal is vertical, and is rotated with constant angular speed about this diagonal. Calculate the angular momentum and torque necessary to sustain this motion with respect to the lowest corner. Express your answer in terms of M, a, b and t (a and b designated in the drawing below.)



Part II - ELECTRICITY AND MAGNETISM

Answer 3 out of 4 questions (5-8)

5. A bar magnet is suspended so that it is free to rotate over the center of a copper disk which lies in a horizontal plane.

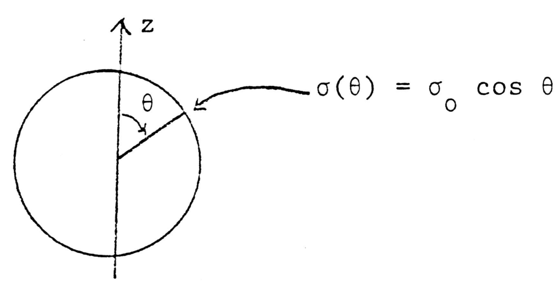


The disk is made to rotate in a counterclockwise direction.

a) Describe qualitatively the pattern of currents induced in the rotating disk.

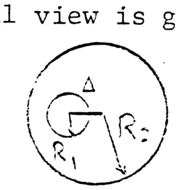
b) Will the magnet rotate in the same or opposite direction as the disk? Why?

6. A charge density $\sigma(\theta) = \sigma_o \cos\theta$ esu/cm^2 is glued over the surface of a spherical shell of radius R (σ_o is a constant and θ is the polar angle as shown in the sketch). There is a vacuum, with no charges, both inside and outside of the shell. Calculate the electrostatic potential and the electric field both inside and outside the shell.



7. A conducting spherical shell of radius a, is placed in a uniform electrical field $\vec{E}$. Find the force tending to separate two halves of the sphere across a diametrical plane perpendicular to $\vec{E}$. Let $|\vec{E}| = E_o$.

8. Consider a capacitor formed out of two (infinitely long) cylinders of radii R_1 and $R_2 > R_1$ which are parallel. Their centers are separated by a distance Δ. A vertical view is given in the figure:



Compute the capacitance C of this capacitor (per unit length). The electric permittivity of the material is constant and equal to ε.